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Estimation of the Mean Value for the Normal Distribution with Constraints on d-Risk

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Abstract—We consider the problem of an estimation of the mean value of the normal distribution with a prior information that this parameter is positive and very small. The prior information is implemented in terms of the exponential prior distribution. The estimation procedures are constructed for two cases: fixed sample size and sequential estimation that guarantee the given constraints on the precision and the d-risk of the estimator. An analytical review of the comprehensive literature for the problems of guaranteed statistical inference (d-risk and pFDR) is provided. For the practical applications of the proposed estimators with the unknown value of the prior distribution parameter, we solve the problem of choosing this parameter in the framework of empirical (parametric) Bayesian approach or in the framework of existing State Standards on the precision and output quality of the estimated parameter. As an implementation of the proposed statistical procedures, the problem of estimation of the chemical element of arsenic (As) in a food product is considered. The model parameters are chosen according to the State Standards for carrying out a laboratory tests for As detection. For the chosen values of the parameters, the probability of stopping for the experiment is estimated for each step by the method of statistical simulations. The histogram of the Bayesian estimate for the As content is presented.

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1. INTRODUCTION

We consider the problem of estimation of the mean value θ of the normal distribution when we have a prior information on this estimated parameter: It is positive and very small. Such statistical problem appears often in laboratory investigations when it is required to estimate a content of some extraneous contaminates in a product. For example, it is required to estimate the presence of heavy metals or arsenic in a food product. Since the true unknown value of θ, which defines the distribution of a random sample, is obviously a realization of a random variable ϑ, we should estimate value θ in the framework of Bayesian approach introducing a prior distribution of ϑ. The information on small values of ϑ says that the prior distribution should have the probability mass concentrated on positive close to zero values of ϑ. We should expect that taking into consideration this information and with the help of estimation of the prior distribution by the archive of previous observations, will reduce the sample size which is needed to estimate θ with required precision and reliability.

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